1995 #1

a) $x^2+x+1>0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

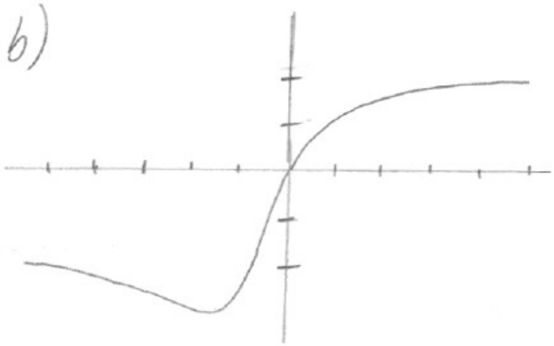
$$=\frac{-1\pm\sqrt{1-4(1)(1)}}{2(1)}$$

$$=\frac{-1\pm\sqrt{-3}}{2}$$

Since it is imaginary under the radical all values will ok for x

$\therefore$ Domain $\in \mathbb{R}$

b)



c) $\lim\limits_{x\to\infty} \frac{2x}{\sqrt{x^2+x+1}}$

$y=\pm 2$

d) $f'(x)=\frac{x+2}{(x^2+x+1)^{\frac{3}{2}}}$

	-2
$x+2$	$- \; - \; 0 \; + \; + \; +$
$(x^2+x+1)^{\frac{3}{2}}$	$+ \; + \; + \; + \; ++$
$f'(x)$	$- \; 0 \; +$

min occurs at $x=-2$

$$f(-2)=\frac{2(-2)}{\sqrt{(-2)^2-2+1}}$$

$$=\frac{-4}{\sqrt{4-1}}=\frac{-4}{\sqrt{3}}$$

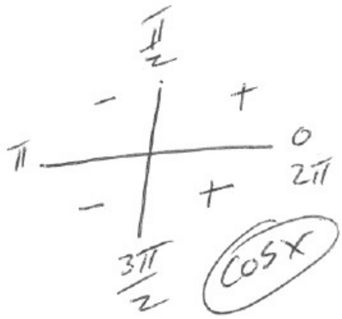
$f(-2)=-2.3094$

The max occurs at $y=2$ since this is the horizontal asymptote and the curve is approaching from below

Range $-2.3094\leq f(x)<2$

2a. $t\cos t \ge 0$

	0		$\frac{\pi}{2}$		$\frac{3\pi}{2}$	
t	0	+ +	+ +	+	+ + +	
$\cos t$		+	0	−	0	+
		+	0	−	0	+ (1)



$$\boxed{\begin{array}{l}\text{Increasing } 0 \le t \le \frac{\pi}{2} \\ \frac{3\pi}{2} \le t \le 5\end{array}}$$ (1) (1)

b. accel = $v'(t)$

$= t(-\sin t) + \cos t$

$\boxed{a(t) = -t\sin t + \cos t}$

2

c. distance $= \int v(t)$ $v = t\cos t$

$uv - \int v\,du$

$u = t$ $dv = -\sin t$ $\cos t$

$du = dt$ $v = \cos t$ $\sin t$

$\cancel{t\cos t - \int \cos t\,dt}$ $t\sin t - \int \sin t\,dt$

$\cancel{= t\cos t - \sin t + c}$ $t\sin t + \cos t + c$

$3 = 0\sin 0 + \cos 0 + c$

$\cancel{3 = 0\cos 0 - \sin 0 + c}$ $3 = 1 + c$ $c = +2$

$2 = c$

dist $= s(t) = \cancel{t\cos t - \sin t + 2}$ $\boxed{t\sin t + \cos t + 2}$ (2)

d. $s(\frac{\pi}{2}) = \frac{\pi}{2}\cos\frac{\pi}{2} - \sin\frac{\pi}{2} + 2$ Velocity = 0 at_

$\cancel{= 2 + \frac{\pi}{2}}$ $\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} + 2$ (2)

$\boxed{= 3.571 = 2 + \frac{\pi}{2}}$

3a) $\frac{dy}{dx} = ?$

$$-16x + 5x\frac{dy}{dx} + 5y + 3y^2\frac{dy}{dx} = 0$$

$$(5x + 3y^2)\frac{dy}{dx} = -5y + 16x \qquad 1$$

$$\frac{dy}{dx} = \frac{-5y + 16x}{5x + 3y^2}$$

b) $\frac{dy}{dx} = \frac{-5(-1) + 16(4)}{5(4) + 3(-1)^2}$

$$= \frac{69}{23} = 3 \qquad 1$$

$$3 = \frac{y+1}{x-4}$$

$$3(x-4) = y+1 \qquad 1$$

c) $m = \frac{y_2 - y_1}{x_2 - x_1}$

USING SLOPE

$$3 = \cancel{\frac{4-4.2}{-1-k}} \quad \frac{-1-k}{4-4.2} \qquad 1$$

$$\cancel{-3-3k = 4-4.2}$$

$$\cancel{-3k=}$$

$$3(-.2) = -1-k$$

$$.4 = -k$$

$$k = -.4 \qquad 1$$

USING DIFFERENTIALS

$$f(x+\Delta x) = f(x) + f'(x)\Delta x$$

$$= f(4) + f'(x)\Delta x$$

$$= -1 + 3(.2)$$

$$= -.4$$

$$3(x-4) = y+1 \leftarrow \text{Eqn from 'b'}$$

$$3(4.2-4) = y+1$$

$$3(.2) = y+1$$

$$-.4 = y$$

3d) $-8x^2+5xy+y^3=-149$

$-8(4.2)^2+5(4.2)k+k^3=-149$ 2

$-141.12+21k+k^3=-149$

3e) 2

$k^3+21k+7.88=0$

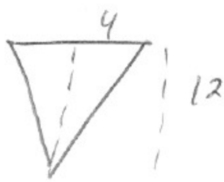
use calculators

$k=-.373$

4a.) $(-.767, .588)$, $(2,4)$, $(4,16)$

b.) $\int_{-.767}^{2}(2^x-x^2)dx+\int_{2}^{4}(x^2-2^x)dx$

c.) $\pi\int_{-.767}^{2}((5-x^2)^2-(5-2^x)^2)dx$

5a) 

$r=\frac{4}{12}h=\frac{h}{3}$

3

$V=\frac{1}{3}\pi r^2h$

$V(h)=\frac{1}{3}\pi\left(\frac{h}{3}\right)^2h$

$=\frac{\pi h^3}{27}$

5b) $\frac{dv}{dt} = \frac{3\pi h^2}{27}\frac{dh}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$

$\frac{dh}{dt} = (h-12) = (3-12)$ 3

$= -9$ ft/min

$\frac{dv}{dt} = \frac{3\pi 3^2(-9)}{27}$

$= -9\pi$ cubic ft/min

5c) Volume = base × height

$V = 400\pi Y$ 3

$\frac{dv}{dt} = 400\pi \frac{dy}{dt}$

$9\pi = 400\pi \frac{dy}{dt}$

$\frac{9\pi}{400\pi} = \frac{9}{400} = \frac{dy}{dt}$ ~~cubic~~ ft/min

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6a) $h(1) = \int_1^1 f(t)\,dt = 0$ 2

b) $h'(4) = 2$ note: the graph represents the derivative of h 2

c) since f represents the derivative of h if f is increasing then it is concave up.

The graph ∴ is concave up in the following intervals $[+1, 3]$ & $[6, 7]$ 3

d) MIN 2 MIN AT $X=1$

$\int_1^5 f(t) > \int_7^5 f(t)$